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TECHNICAL NOTES

EXACT ANALYSIS OF CONVECTIVE DIFFUSION OF A SOLUTE IN MHD RADIATING CONVECTIVE FLOW IN A VERTICAL CHANNEL

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NOMENCLATURE

B_0 applied magnetic field
C concentration of the solute, $C(t, x, y)$
C^* constant related to radiation appearing in ref. [4] as C
C_4^* constant defined by equation (1)
F radiation parameter
g gravitational acceleration
L half-width of channel
M Hartmann number
N vertical temperature gradient
Ra Rayleigh number.

Greek symbols
α thermal diffusivity
β volumetric expansion coefficient
μ magnetic permeability
ν kinematic viscosity
σ electrical conductivity
ρ reference density.

INTRODUCTION

THE STUDY of hydromagnetic convection with heat transfer is very important in the design of MHD generators, cross-field accelerators, shock tubes, pumps and cooling systems of reactors and many other fields of technology. For this reason such flows have been investigated by several authors. A comprehensive review of these works has been given in ref. [1]. Gershuni and Zhukhovitskii [2] investigated convective MHD flow in a vertical channel when the wall temperatures were constant while Yu [3] investigated the same problem when the plate temperature varies linearly with vertical distance. All these authors took a transverse magnetic field and neglected heat transfer by radiation. But for similar problems concerned with space application as well as those for which operating temperatures are very high, heat transfer by radiation should not be neglected. Gupta and Gupta [4] extended the problem of Yu [3] to include the radiation effect. For measuring flow rate, velocity etc. tracer elements were introduced into the flows and the study of dispersion of these solutes introduced is therefore very important. Such studies were initiated by Taylor [5, 6], and Aris [7]. Mazumdar [8] studied Taylor diffusion for a natural convective flow through a vertical channel when the plate temperature varies linearly with vertical distance. However, he took neither magnetic field nor radiation into account. Mandal *et al.* [9] extended the work of Mazumdar [8] to include the effect of radiation as well as the magnetic field taking the fluid to be electrically conducting. Both Mazumdar [8] and Mandal *et al.* [9] worked out long time analysis of their problems following the work of Aris [7].

The aim of this note is to study an exact analysis of the unsteady dispersion of a solute in a convective radiating MHD flow of a fluid through a vertical channel with linearly varying wall temperatures using the Gill and Sankarasubramanian [10] model. This analysis is valid for all time.

ANALYSIS

We consider the steady laminar free and forced convective flow of an electrically conducting incompressible viscous fluid between two infinite electrically non-conducting parallel vertical plates $y = \pm L$ in the presence of a uniform transverse magnetic field B_0 along the y-axis when the wall temperature varies linearly with vertical distance. The velocity profile for such a flow has been derived by Gupta and Gupta [4] in the form

$$u_x = \frac{\alpha C_4^*}{L(M^2F+Ra)}\left[F - \frac{k_2^2(F-k_1^2)}{(k_2^2-k_1^2)}\frac{\cosh k_1\eta}{\cosh k_1} + \frac{k_1^2(F-k_2^2)}{(k_2^2-k_1^2)}\frac{\cosh k_2\eta}{\cosh k_2}\right], \quad (1)$$

where

$$k_1 = [(F+M^2)/2 + \{(F-M^2)^2 - 4\,Ra\}^{1/2}/2]^{1/2},$$
$$k_2 = [(F+M^2)/2 - \{(F-M^2)^2 - 4\,Ra\}^{1/2}/2]^{1/2}, \quad (2)$$
$$F = L^2C^*/\alpha, \quad M = B_0L(\sigma/\rho\nu)^{1/2}, \quad Ra = g\beta NL^4/\nu\alpha.$$

If a solute diffuses in the above fully developed flow, the concentration $C(t, x, y)$ of the solute satisfies the equation

$$\frac{\partial C}{\partial t} + u_x\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right). \quad (3)$$

We introduce the dimensionless variables

$$\theta = \frac{C}{C_0}, \quad X = \frac{Dx}{L^2\bar{u}}, \quad \tau = \frac{Dt}{L^2},$$
$$\eta = \frac{y}{L}, \quad X_s = \frac{Dx_s}{L^2\bar{u}}, \quad Pe = \frac{\bar{u}L}{D}, \quad (4)$$

where

$$\bar{u}\left(= \frac{1}{2L}\int_{-L}^{L} u_x\,\mathrm{d}y\right)$$

is the mean velocity and C_0 is the concentration of the initial slug input satisfying

$$C(0, x, y) = C_0 \quad \text{for } |x| \leqslant \tfrac{1}{2}x_s,$$

and (5)

$$C(0, x, y) = 0 \quad \text{for } |x| > \tfrac{1}{2}x_s.$$

We now define a new axial coordinate moving with the average velocity $\bar{u}$ of the flow as $x_1 = x - \bar{u}t$, i.e. $\xi = X - \tau$ in dimensionless form.

Using equations (1) and (4) in equation (3) and transforming to the (τ, ξ, η) coordinate system, we obtain

$$\frac{\partial \theta}{\partial \tau}+\frac{u_x-\bar{u}}{\bar{u}}\frac{\partial \theta}{\partial \xi}=\frac{1}{Pe^2}\frac{\partial^2 \theta}{\partial \xi^2}+\frac{\partial^2 \theta}{\partial \eta^2}. \tag{6}$$

The boundary and initial conditions for equation (6) are

$$\begin{aligned}&\frac{\partial \theta}{\partial \eta}(\tau, X, -1)=\frac{\partial \theta}{\partial \eta}(\tau, X, +1)=0,\\ &\theta(\tau, \infty, \eta)=0,\\ &\theta(0, X, \eta)=1 \quad \text{for } |X| \leqslant \tfrac{1}{2}X_s,\\ \text{and}\quad &\theta(0, X, \eta)=0 \quad \text{for } |X| > \tfrac{1}{2}X_s,\end{aligned} \tag{7}$$

where the first two conditions are consistent with no mass transfer at the channel walls.

We now assume that the solution of equation (6) can be written in the form

$$\begin{aligned}\theta &= \theta_m(\tau,\xi)+\sum_{k=1}^{\infty} f_k(\tau,\eta)\frac{\partial^k \theta_m}{\partial \xi^k},\\ \theta_m &= \frac{1}{2}\int_{-1}^{1}\theta \, d\eta.\end{aligned} \tag{8}$$

Then following Gill and Sankarasubramanian [10], we assume that the process of distributing θ_m is diffusive in nature right from time zero (unlike that in the Taylor model) and introduce the generalized dispersion model with the time-dependent dispersion coefficient as

$$\frac{\partial \theta_m}{\partial \tau}=\sum_{i=1}^{\infty} K_i(\tau)\frac{\partial^i \theta_m}{\partial \xi^i}. \tag{9}$$

From equations (6)–(9) we get $K_1(\tau) = 0$ and

$$\begin{aligned}K_2(\tau) = {} & \frac{1}{Pe^2}+\frac{A^2}{2}\Bigg[k_2^4(F-k_1^2)^2\left\{\left(\frac{\tanh k_1}{k_1}\right)^2\left(\frac{5}{3}+\frac{4}{k_1^2}\right)-\frac{3}{k_1^2}\frac{\tanh k_1}{k_1}-\frac{1}{k_1^2}\right\}\\ & +k_1^4(F-k_2^2)^2\left\{\left(\frac{\tanh k_2}{k_2}\right)^2\left(\frac{5}{3}+\frac{4}{k_2^2}\right)-\frac{3}{k_2^2}\frac{\tanh k_2}{k_2}-\frac{1}{k_2^2}\right\}\\ & -k_1^2k_2^2(F-k_1^2)(F-k_2^2)\Bigg(4\frac{\tanh k_1 \tanh k_2}{k_1k_2}\left(\frac{1}{3}+\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)-\frac{2}{k_2^2}\frac{\tanh k_1}{k_1}\\ & -\frac{2}{k_1^2}\frac{\tanh k_2}{k_2}-\frac{2}{(k_2^2-k_1^2)}\left\{\frac{\tanh k_2}{k_2}\left(1+\frac{k_2^2}{k_1^2}\right)-\frac{\tanh k_1}{k_1}\left(1+\frac{k_1^2}{k_2^2}\right)\right\}\Bigg)\Bigg]\\ & -2k_1^4k_2^4A^2\sum_{n=1}^{\infty}\frac{1}{n^2\pi^2}\left[\frac{(F-k_1^2)}{(k_1^2+n^2\pi^2)}\frac{\tanh k_1}{k_1}-\frac{(F-k_2^2)}{(k_2^2+n^2\pi^2)}\frac{\tanh k_2}{k_2}\right]^2 e^{-n^2\pi^2\tau},\end{aligned} \tag{10}$$

where

$$A = 1\Bigg/\left[(k_2^2-k_1^2)F-k_2^2(F-k_1^2)\frac{\tanh k_1}{k_1}+k_1^2(F-k_2^2)\frac{\tanh k_2}{k_2}\right]. \tag{11}$$

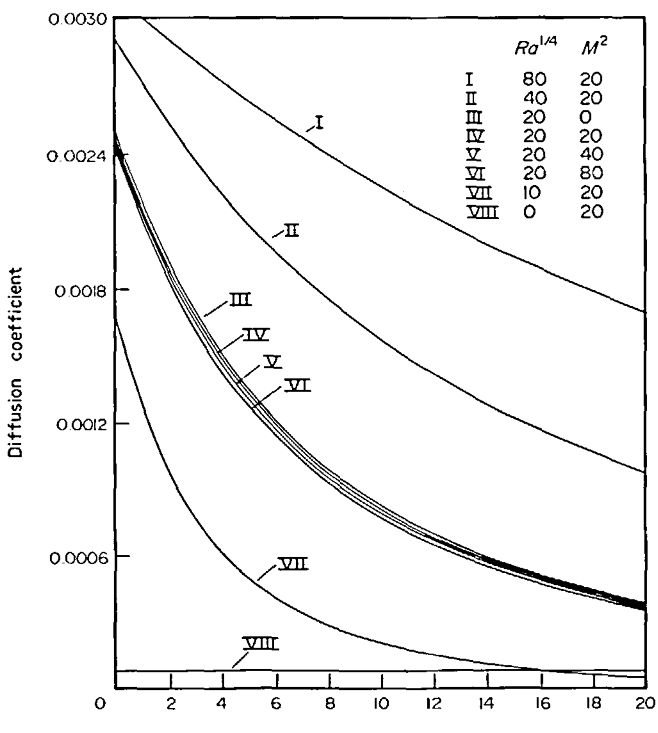

FIG. 1.

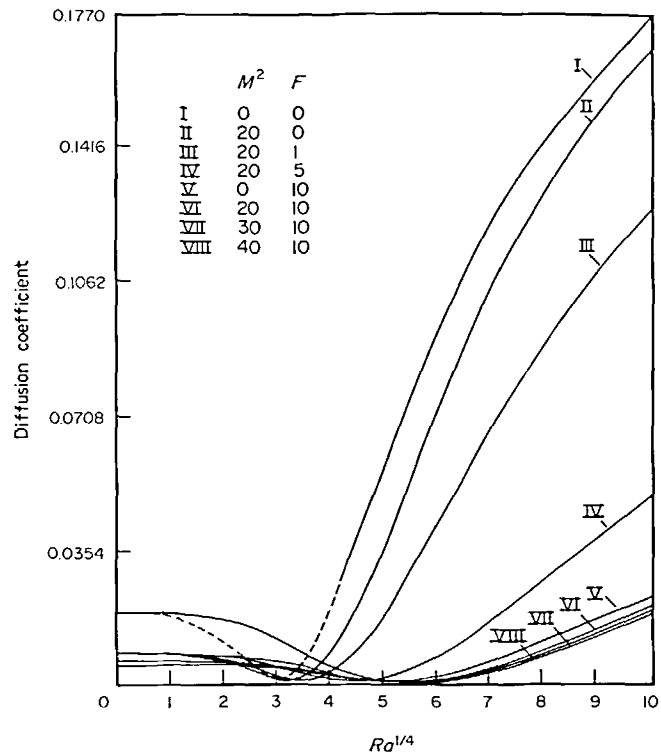

FIG. 2.

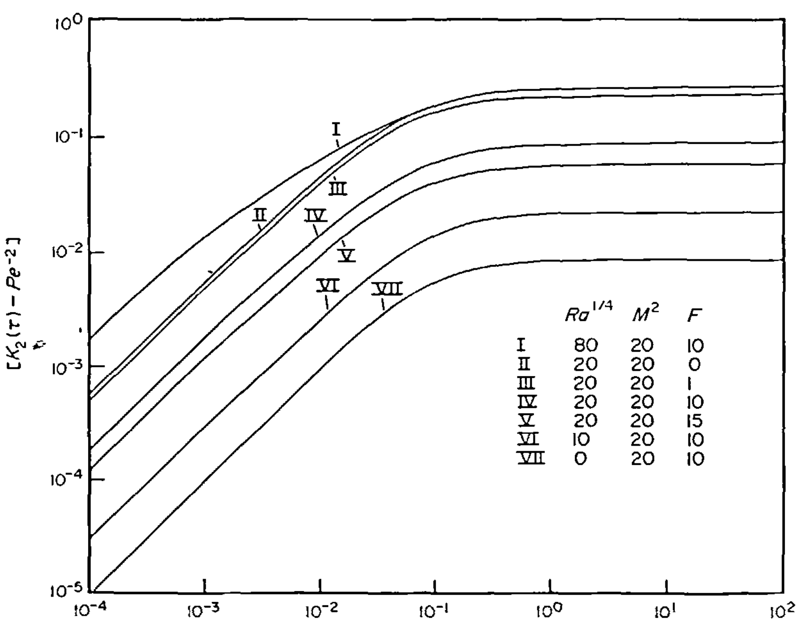

FIG. 3. Plot of $[K_2(\tau) - Pe^{-2}]$ vs τ for several values of M^2, F and Ra.

RESULTS AND DISCUSSIONS

We assume that, as in other cases $K_3(\tau)$ is much smaller than $K_2(\tau)$ in this case also, and thus we confined ourselves to the discussion on $K_2(\tau)$ only.

In the asymptotic case when $\tau \to \infty$ we have calculated the diffusion coefficient $[K_2(\tau)-Pe^{-2}]$ [for various values of the Hartmann number (M), radiation parameter (F) and Rayleigh number (Ra)] and have plotted them against F and $Ra^{1/4}$, respectively, in Figs. 1 and 2. It is interesting to note that these graphs coincide with the corresponding graphs of the effective Taylor diffusion coefficient calculated with the Taylor model by Mandal *et al.* [9]. This further strengthens the use of the Gill and Sankarasubramanian model and treats $K_2(\tau)-Pe^{-2}$ as the diffusion coefficient. Following Kay [11], and Grief *et al.* [12] we worked out all calculations taking $Ra^{1/4}$ in the range 0–10^2.

Figure 1 shows that the diffusion coefficient is very sensitive to any change in the value of $Ra^{1/4}$ independently of the value of F and M namely it decreases rapidly with a decrease in $Ra^{1/4}$.

Figure 2 shows that the diffusion coefficient has a minimum value at $Ra^{1/4} = r_m$ where r_m lies between 0 and 10 depending on the value of F. When $Ra^{1/4} > r_m$ a change in F drastically changes the diffusion coefficient. However, when $Ra^{1/4} < r_m$ the effect of F is not so significant. Thus in the experimentally observed range (10–10^2) as indicated by Kay [11] the role of the radiation parameter F is very important. However, it is clear from both Figs. 1 and 2 that with an increase in radiation the diffusion coefficient decreases. This is due to the fact that due to loss of energy by radiation the velocity decreases and thereby the diffusion coefficient also decreases. Figures 1 and 2 both show that the role of M is not very significant in comparison with those of $Ra^{1/4}$ and F.

In Figure 3 we have plotted $[K_2(\tau)-Pe^{-2}]$ against τ for various values of F, $Ra^{1/4}$ and M. The asymptotic value of $[K_2(\tau)-Pe^{-2}]$ is reached at a time τ of $O(1)$ and this attainment is independent of all parameters. In the initial stage the effects of the parameters are similar to those in the asymptotic case.

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EFFECT OF CROSSFLOW IN BOILING HEAT TRANSFER OF WATER

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NOMENCLATURE

D	outside diameter of tube [m]
G	mass velocity [kg s^{-1} m^{-2}]
h	heat transfer coefficient [W m^{-2} K^{-1}]
k	thermal conductivity of fluid [W m^{-1} K^{-1}]
n	index in the Kutateladze equation (1)
Pr	Prandtl number of fluid
q	heat flux [W m^{-2}]
T	temperature [K]
ΔT	wall superheat [K].

Greek symbol

μ	dynamic viscosity of fluid [kg m^{-1} s^{-1}].

Subscripts

b	pool boiling
F	condition at mean film temperature
f	forced convection
fb	forced convection boiling
s	saturation condition
w	heated surface condition.

INTRODUCTION

Crossflow boiling has not been adequately investigated. A few investigators [1–4] who have worked on crossflow boiling have experimentally studied the phenomenon with water at